Limits & Derivatives

Evaluate :

1. $\lim_{x\to 3}\frac{x^2-9}{x-3}$
2. $\lim_{x\to \frac{1}{2}}\frac{4x^2-1}{2x-1}$
3. $\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$
4. $\lim_{x\to 0}\frac{(x+2)^{\frac{1}{3}}-2^{\frac{1}{3}}}{x}$
5. $\lim_{x\to 1}\frac{(1+x)^6-1}{(1+x)^2-1}$
6. $\lim_{x\to a}\frac{(2+x)^{\frac{5}{2}}-(a+2)^{\frac{5}{2}}}{x-a}$
7. $\lim_{x\to 1}\frac{x^4-\sqrt{x}}{\sqrt{x}-1}$
8. $\lim_{x\to 2}\frac{x^2-4}{\sqrt{3x-2}-\sqrt{x+2}}$
9. $\lim_{x\to\sqrt{2}}\frac{x^4-4}{x^2+3\sqrt{2}x-8}$
10. $\lim_{x\to 1}\frac{x^7-2x^5+1}{x^3-3x^2+2}$
11. $\lim_{x\to 0}\frac{\sqrt{1+x^3}-\sqrt{1-x^3}}{x^2}$
12. $\lim_{x\to -3}\frac{x^3+27}{x^5+243}$
13. $\lim_{x\to \frac{1}{2}}\left(\frac{8x-3}{2x-1}-\frac{4x^2+1}{4x^2-1}\right)$
14. Find 'n', if $\lim_{x\to 2}\frac{x^n-2^n}{x-2}=80$, $n\in \mathbf{N}$
15. $\lim_{x\to a}\frac{\sin 3x}{\sin 7x}$
16. $\lim_{x\to 0}\frac{\sin^2 2x}{\sin^2 4x}$
17. $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$
18. $\lim_{x\to 0}\frac{2\sin x-\sin 2x}{x^3}$
19. $\lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx}$
20. $\lim_{x\to \frac{\pi}{3}}\frac{\sqrt{1-\cos 6x}}{\sqrt{2}\left(\frac{\pi}{3}-x\right)}$
21. $\lim_{x\to \frac{\pi}{4}}\frac{\sin x-\cos x}{x-\frac{\pi}{4}}$
22. $\lim_{x\to \frac{\pi}{6}}\frac{\sqrt{3}\sin x-\cos x}{x-\frac{\pi}{6}}$
23. $\lim_{x\to 0}\frac{\sin 2x+3x}{2x+\tan 3x}$
24. $\lim_{x\to a}\frac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}$
25. $\lim_{x\to \frac{\pi}{6}}\frac{\cot^2 x-3}{\operatorname{cosec} x-2}$
26. $\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$

27. $\lim_{x\to 0}\frac{\sin x-2\sin 3x+\sin 5x}{x}$

28. If $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, then find the value of k.

Differentiate each of the functions w. r. to x in Exercises 29 to 42.

29. $\frac{x^4+x^3+x^2+1}{x}$ 30. $\left(x+\frac{1}{x}\right)^3$ 31. $(3x+5)(1+\tan x)$

32. $(\sec x-1)(\sec x+1)$ 33. $\frac{3x+4}{5x^2-7x+9}$ 34. $\frac{x^5-\cos x}{\sin x}$

35. $\frac{x^2\cos\frac{\pi}{4}}{\sin x}$ 36. $(ax^2+\cot x)(p+q\cos x)$

37. $\frac{a+b\sin x}{c+d\cos x}$ 38. $(\sin x+\cos x)^2$ 39. $(2x-7)^2(3x+5)^3$

40. $x^2\sin x+\cos 2x$ 41. $\sin^3 x\cos^3 x$ 42. $\frac{1}{ax^2+bx+c}$

Differentiate each of the functions with respect to 'x' in Exercises 43 to 46 using first principle.

43. $\cos(x^2+1)$ 44. $\frac{ax+b}{cx+d}$ 45. $x^{\frac{2}{3}}$

46. $x\cos x$

Evaluate each of the following limits in Exercises 47 to 53.

47. $\lim_{y\to 0}\frac{(x+y)\sec(x+y)-x\sec x}{y}$

48. $\lim_{x\to 0} \frac{(\sin(\alpha+\beta)x + \sin(\alpha-\beta)x + \sin 2\alpha x)}{\cos 2\beta x - \cos 2\alpha x} \cdot x$

49. $\lim_{x\to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x+\frac{\pi}{4}\right)}$

50. $\lim_{x\to \pi} \frac{1-\sin\frac{x}{2}}{\cos\frac{x}{2}\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)}$

51. Show that $\lim_{x\to 4} \frac{|x-4|}{x-4}$ does not exists

52. Let $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x} & \text{when } x \neq \frac{\pi}{2} \\ 3 & x = \frac{\pi}{2} \end{cases}$ and if $\lim_{x\to \frac{\pi}{2}} f(x) = f(\frac{\pi}{2})$,

find the value of k

53. Let $f(x) = \begin{cases} x+2 & x \leq -1 \\ cx^2 & x > -1 \end{cases}$, find '$c$' if $\lim_{x\to -1} f(x)$ exists.

Objective Type Questions

Choose the correct answer out of 4 options given against each Exercise 54 to 76 (M.C.Q).

54. $\lim_{x\to \pi} \frac{\sin x}{x-\pi}$ is

(A) 1 (B) 2 (C) –1 (D) –2

55. $\lim_{x\to 0} \frac{x^2 \cos x}{1-\cos x}$ is

(A) 2 (B) $\frac{3}{2}$ (C) $\frac{-3}{2}$ (D) 1

56. $\lim_{x\to 0}\frac{(1+x)^n-1}{x}$ is

(A) n (B) 1 (C) $-n$ (D) 0

57. $\lim_{x\to 1}\frac{x^m-1}{x^n-1}$ is

(A) 1 (B) $\frac{m}{n}$ (C) $-\frac{m}{n}$ (D) $\frac{m^2}{n^2}$

58. $\lim_{x\to 0}\frac{1-\cos 4\theta}{1-\cos 6\theta}$ is

(A) $\frac{4}{9}$ (B) $\frac{1}{2}$ (C) $\frac{-1}{2}$ (D) –1

59. $\lim_{x\to 0}\frac{\operatorname{cosec} x-\cot x}{x}$ is

(A) $\frac{-1}{2}$ (B) 1 (C) $\frac{1}{2}$ (D) 1

60. $\lim_{x\to 0}\frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$ is

(A) 2 (B) 0 (C) 1 (D) –1

61. $\lim_{x\to\frac{\pi}{4}}\frac{\sec^2 x-2}{\tan x-1}$ is

(A) 3 (B) 1 (C) 0 (D) $\sqrt{2}$

62. $\lim_{x\to 1}\frac{(\sqrt{x}-1)(2x-3)}{2x^2+x-3}$ is

(A) $\frac{1}{10}$ (B) $\frac{-1}{10}$ (C) 1 (D) None of these

63. If $f(x)=\begin{cases}\frac{\sin[x]}{[x]}, [x]\neq 0\\ 0 \quad , [x]=0\end{cases}$, where [.] denotes the greatest integer function, then $\lim_{x\to 0} f(x)$ is equal to

(A) 1 (B) 0 (C) –1 (D) None of these

64. $\lim_{x\to 0}\frac{|\sin x|}{x}$ is

(A) 1 (B) –1 (C) does not exist (D) None of these

65. Let $f(x)=\begin{cases}x^2-1, 0<x<2\\ 2x+3, 2\le x<3\end{cases}$, the quadratic equation whose roots are $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$ is

(A) $x^2-6x+9=0$ (B) $x^2-7x+8=0$
(C) $x^2-14x+49=0$ (D) $x^2-10x+21=0$

66. $\lim_{x\to 0}\frac{\tan 2x-x}{3x-\sin x}$ is

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{-1}{2}$ (D) $\frac{1}{4}$

67. Let $f(x)=x-[x]; \in \mathbf{R}$, then $f'\left(\frac{1}{2}\right)$ is

(A) $\frac{3}{2}$ (B) 1 (C) 0 (D) –1

68. If $y=\sqrt{x}+\frac{1}{\sqrt{x}}$, then $\frac{dy}{dx}$ at $x=1$ is

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) 0

69. If $f(x) = \frac{x-4}{2\sqrt{x}}$, then $f'(1)$ is

(A) $\frac{5}{4}$ (B) $\frac{4}{5}$ (C) 1 (D) 0

70. If $y = \frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}}$, then $\frac{dy}{dx}$ is

(A) $\frac{-4x}{(x^2-1)^2}$ (B) $\frac{-4x}{x^2-1}$ (C) $\frac{1-x^2}{4x}$ (D) $\frac{4x}{x^2-1}$

71. If $y = \frac{\sin x + \cos x}{\sin x - \cos x}$, then $\frac{dy}{dx}$ at $x = 0$ is

(A) –2 (B) 0 (C) $\frac{1}{2}$ (D) does not exist

72. If $y = \frac{\sin(x+9)}{\cos x}$ then $\frac{dy}{dx}$ at $x = 0$ is

(A) cos 9 (B) sin 9 (C) 0 (D) 1

73. If $f(x) = 1 + x + \frac{x^2}{2} + \ldots + \frac{x^{100}}{100}$, then $f'(1)$ is equal to

(A) $\frac{1}{100}$ (B) 100 (C) does not exist (D) 0

74. If $f(x) = \frac{x^n - a^n}{x-a}$ for some constant 'a', then $f'(a)$ is

(A) 1 (B) 0 (C) does not exist (D) $\frac{1}{2}$

75. If $f(x) = x^{100} + x^{99} + \ldots + x + 1$, then $f'(1)$ is equal to

(A) 5050 (B) 5049 (C) 5051 (D) 50051

76. If $f(x) = 1 - x + x^2 - x^3 \ldots - x^{99} + x^{100}$, then $f'(1)$ is euqal to

(A) 150 (B) –50 (C) –150 (D) 50